Eddy Currents

Objectives

After going through this lesson, the learners will be able to:

- Understand Eddy Currents and demonstrate the same through simple experiments.
- Identify the effects of eddy currents and learn methods to reduce or enhance their effect for useful purposes.
- Comprehend the concept of self and mutual induction
- Define coefficient of mutual induction for a pair of long co-axial solenoids and obtain the mathematical expression for the same
- List the factors on which the coefficient of self-inductance and mutual induction of a pair of coils depends.
- Appreciate the reasons for referring to the self-induced emf as 'back emf'
- Obtain an expression for the energy required to build up a current I in a coil of self-inductance L.
- Define the SI unit of inductance – the henry (H) in different ways

Content Outline

- Unit Syllabus
- Module Wise Distribution of Unit Syllabus
- Words You Must Know
- Introduction
- Eddy Currents
- Experimental Demonstration of Eddy Currents
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- Energy Required to Build up a Current Through a Solenoid
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- Coefficient of Mutual Induction

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Unit Syllabus

Unit IV: Electromagnetic Induction and Alternating Currents

Chapter-6: Electromagnetic Induction

Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law, Eddy currents.

Self and mutual induction.

Chapter-7: Alternating Current

Alternating currents, peak and rms value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, wattless current. AC generator and transformer.

Module Wise Distribution of Unit Syllabus - 09 Modules

The above unit is divided into 9 modules for better understanding.

Module 1	● Electromagnetic induction ● Faraday's laws, induced emf and current; ● Change of flux ● Rate of change of flux
Module 2	● Lenz's Law, ● Conservation of energy ● Motional emf
Module 3	● Eddy currents ● Self induction ● Mutual induction ● Unit ● Numerical
Module 4	● AC generator ● Alternating currents, ● Representing ac ● Formula

	• Graph • Phasor • Frequency of ac and what does it depend upon • Peak and rms value of alternating current/voltage;
Module 5	• AC circuits • Components in ac circuits • Comparison of circuit component in ac circuit with that if used in dc circuit • Reactance mathematically • Pure R • Pure L • Pure C • Phasor, graphs for each
Module 6	• AC circuits with RL, RC and LC components • Impedance; LC oscillations (qualitative treatment only), • Resonance • Quality factor
Module 7	• Alternating voltage applied to series LCR circuit • Impedance in LCR circuit • Phasor diagram • Resonance • Power in ac circuit • Power factor • Wattles current
Module 8	• Transformer
Module 9	• Advantages of ac over dc • Distribution of electricity to your home

Module 3

Words You Must Know

Let us remember the words we have been using in our study of this physics course:

- **Magnetic Field:** The region around a magnet, within which its influence can be felt.

- **Electromotive Force:** The amount of work done by an external source, to take a unit positive charge once round the circuit.
- **Area Vector:** A vector perpendicular to a given area whose magnitude is equal to the given area.
- **Electromagnetic Induction:** An emf is induced in a conductor whenever the magnetic flux around it changes. Due to this phenomenon a current can be generated in a coil by varying magnetic fields.
- **Magnetic Flux:** Just like electric flux, magnetic flux $Ø_B$ through any surface of area **A** held perpendicularly in magnetic field **B** is given by the total number of magnetic lines of force crossing the area. Mathematically, it is equal to the dot product of **B** and **A**.

$\Phi_B = \mathbf{B} . \mathbf{A} = BA \cos \theta$, where θ is the angle between **B** and **A**

- **Induced EMF and Induced Current:** The emf developed in a loop when the magnetic flux linked with it changes with time is called induced emf when the conductor is in the form of a closed loop or is a bulk material, the current induced in the loop is called an induced current.
- **Weber:** One weber is defined as the amount of magnetic flux, through an area of $1m^2$ held normal to a uniform magnetic field of one tesla. The SI unit of magnetic flux is weber (Wb) or Tesla metre squared (Tm^2).
- **Faraday's Laws of Electromagnetic Induction**
  - **First law:** It states that whenever the amount of magnetic flux linked with the coil changes with time, an emf is induced in the coil. The induced emf lasts in the coil only as long as the change in the magnetic flux continues.
  - **Second law:** It states that the magnitude of the emf induced in the coil is directly proportional to the time rate of change of the magnetic flux linked with the coil.

- **Lenz's Law:** The law states that the direction of induced emf is always such that it opposes the change in magnetic flux responsible for its production.
- **Fleming's Right Hand Rule:** Fleming's right hand rule gives us the direction of induced emf / current in a conductor moving in a magnetic field.
- If we stretch the fore-finger, central finger and thumb of our right hand mutually perpendicular to each other such that the fore-finger is in the direction of the field**, the**

thumb is in the direction of motion of the conductor, then the central finger would give the direction of the induced current.

- **Induced EMF by Changing the Magnetic Field:** The movement of magnet or pressing the key of coil results in changing the magnetic field associated with the coil, this induces the emf.
- **Induced EMF by Changing the Orientation of Coil and Magnetic Field:** When the coil rotates in a magnetic field the angle θ changes and magnetic flux linked with the coil changes and this induces the emf. This is the basis of ac generators.
- **Induced EMF by Changing The Area A (MOTIONAL EMF):** Motional emf is a type of induced emf which occurs when a wire is pulled through the magnetic field. The magnitude of motional emf depends upon the velocity of the wire, strength of magnetic field and the length of the wire.
- Motional emf arises due to the motion of charges due to a magnetic field.

Introduction

We have studied so far that current is induced, in closed loops when magnetic flux, linked with them, changes with time.

A thought that arises is : can such currents be induced in bulk pieces of a metal? If yes, what will be the implications of such induced currents wherever metals are being used in regions where there is a changing magnetic field?

We will also study the concept of induction in a single long solenoid due to changing current in itself, which in turn changes the associated magnetic field around it, and in two long solenoids that are in (close) vicinity of each other and if the current in one of them changes.

Eddy Currents

When bulk pieces of conductors are subjected to a changing magnetic flux then induced currents are produced in them. The currents, which are induced in bulk pieces of conductor, when the magnetic flux linked with them changes, are called **Eddy Currents**.

https://commons.wikimedia.org/wiki/File:Mckinney_lower_falls.jpg

The flow patterns of such induced currents, (in bulk pieces of conductors) can resemble swirling eddies in water (**marked in red in the picture).** It is for this reason that these currents are called eddy currents.

This effect was discovered by physicist **Foucault (1819-1868)**; hence these currents are also called **Foucault currents**.

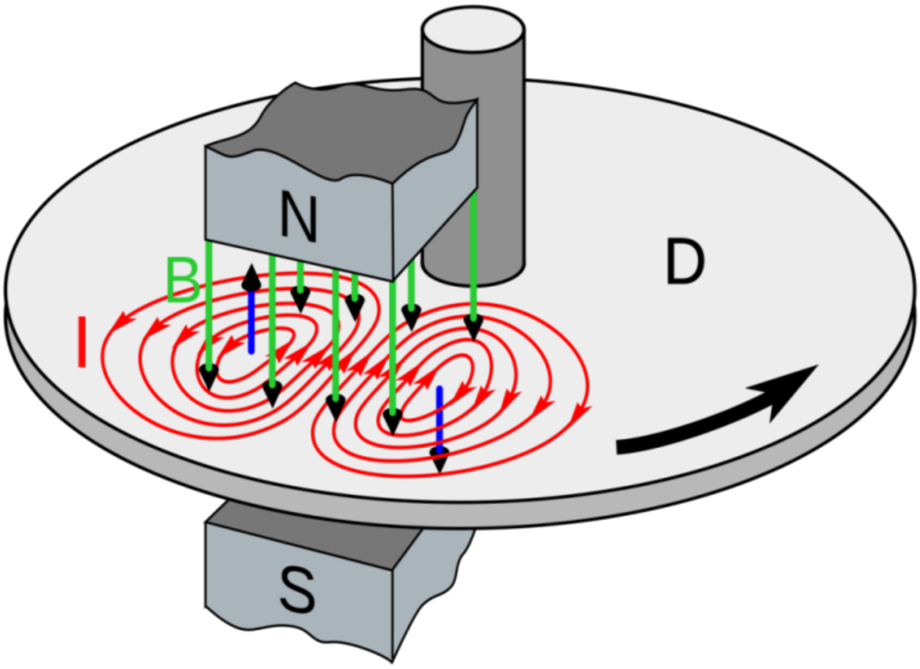
There are delightful videos by Arvind Gupta toys from trash

https://www.youtube.com/watch?v=Y8ru1CMgJM0

https://www.youtube.com/watch?v=g3iDozPvXxo

https://www.youtube.com/watch?v=as4qAMobyS0

There are easy toys you can make on your own to demonstrate eddy currents .



Source: https://commons.wikimedia.org/wiki/File:Eddy_current_brake_diagram.svg

The figure shows a horizontally placed metallic disc between the poles of a horse shoe magnet. The disc will tend to stop quicker than anticipated due to eddy currents in the disc which tend to oppose the cause of development of eddy currents in the disc.

If the magnetic field is absent, or the disc is stationary there would be no eddy currents produced as the magnetic field around the disc would not change hence, no electromagnetic induction would take place.

The magnitude of eddy currents is given by:

$$i = \mathcal{E}/R \quad , \quad e = \frac{d\emptyset}{dt}$$

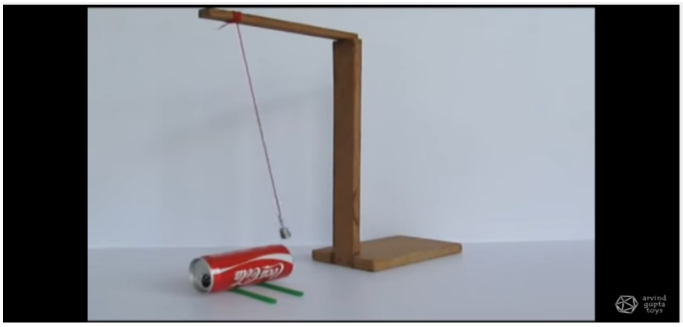
$$Current\ I = \frac{\frac{d\emptyset}{dt}}{R}$$

The direction of an eddy current is given by Lenz's law, but it is not easy to predict the path of the current in bulk matter.

Imagine in the absence of wired circuits the eddy current would flow in loops that could be 3 dimensional.

https://www.youtube.com/watch?v=SF4xjO2RN1w

Arvind Gupta toys



Experimental Demonstration of Eddy Currents

Eddy currents refer to the 'circular currents' induced in a bulk piece of metal, present in a region of changing magnetic field. Their direction is given by Lenz's law.

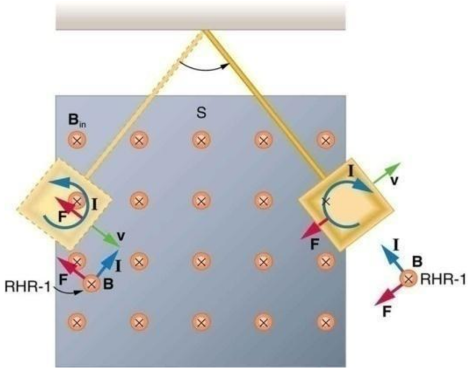
The 'eddy currents' produce a magnetic field which opposes the change in flux which created them. This results in an often undesirable conversion of mechanical energy into thermal energy.

An example of eddy currents is the following: A swinging pendulum, with a conducting sheet, slows down every time it passes through a magnetic field; this ‘slowing’ is due to the ‘eddy currents’ induced in the conducting sheet.

Eddy currents can produce significant drag, called ‘electro-magnetic damping’, on the motion involved.

This can be understand as follows:

Consider the apparatus shown in the figure. Here, a ‘pendulum bob’ swings between the poles of a strong magnet. The ‘pendulum bob’ is a metal sheet, it experiences a force opposing its motion. As it enters from the left, flux increases, and so an eddy current is set up (Faraday's law) in the counterclockwise direction (Lenz' law), as shown.



Source:
https://phys.libretexts.org/TextMaps/Introductory_Physics_Textmaps/Map%3A_College_Physics_(OpenStax)/23%3A_Electromagnetic_Induction%2C_AC_Circuits%2C_and_Electrical_Technologies/23.5_Eddy_Currents_and_Magnetic_Damping

Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (right hand rule). When the metal plate is completely inside the field, there are no eddy currents, (When the field is uniform), since the flux remains constant in this region. When the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left, shows that its motion is damped when entering and leaving the field.

We can explain this phenomenon on the basis of electromagnetic induction. Magnetic flux associated with the plate keeps on changing as the plate moves in and out of the region between magnetic poles.

- **The flux change induces eddy currents in the plate.**
- **Directions of eddy currents are opposite when the plate swings into the region between the poles and when it swings out of the region**.

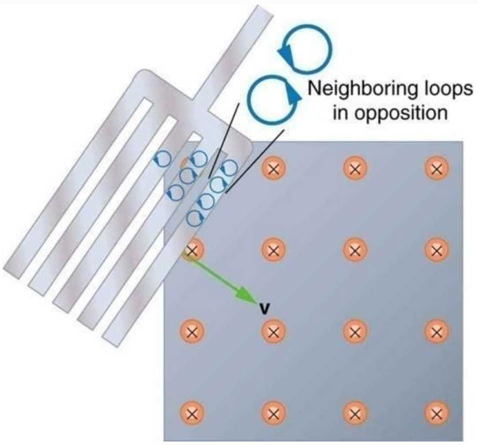
Eddy currents can be reduced by cutting slots into the conductor which minimize the size of possible eddy current loops.

When a slotted metal plate enters the field, as shown in the figure, an EMF is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, hence magnetic damping on insulators is negligible.

Think About This

If eddy currents are to be avoided in conductors,

- **They can be either slotted or**
- **Constructed out of thin layers of conducting material separated by insulating sheets.**



Source: Boundless. “Back EMF, Eddy Currents, and Magnetic Damping.” Boundless Physics. Boundless, 26 May. 2016. Retrieved 17 Jun. 2016 from **Electromagnetic Damping**

https://www.youtube.com/watch?v=DgHjcaYivWE

Arvind Gupta toys

Take a look at the following links to understand eddy currents better.

Eddy Current Experiment With a CD:

In the video you saw that due to the motion of magnets placed on the CD there is a changing magnetic flux associated with it and with the coins placed on top the coins acquire the same polarity as the magnets and hence are repelled .This results in the relative motion between the two .When a bulk metal is placed on top the induced eddy currents oppose the cause of their induction; hence the CD stops.

https://www.youtube.com/watch?v=L8IY4Yi_WIY

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https://www.youtube.com/watch?v=9CH0h8T2Xd8

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PLACE UNDER
CEILING FAN.
AIR WILL SPIN
MAGNETS. THE
ALUMINUM CANS
WILL SPIN
BECAUSE OF
EDDY CURRENTS

Applications of Eddy Currents

Eddy currents are used to advantage, in certain applications like:

(i) Magnetic braking in trains

Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.

(ii) Electromagnetic damping

Certain galvanometers have a fixed core made of non-magnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

To have a closer look at how electromagnetic damping and magnetic brakes work let us take a look at the following clip:
electromagnetic damping/breaking

We saw that the magnet takes much longer in case of its fall through the copper pipe. Why is it so? It is due to the eddy currents that are generated in the copper pipe which oppose their cause, the change in magnetic flux, i.e., the motion of the magnet. The retarding force, due to the eddy currents, inhibits the motion of the magnet. Such phenomena are referred to as **Electromagnetic Damping.**

Note that
Eddy currents will not be generated in PVC pipe as its material is an insulator whereas copper is a conductor.

(iii) Induction Furnace

Induction furnace can be used to produce high temperatures; it can be utilized to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures that are sufficient to melt it.

Let's take a look at how it actually works:

induction heating

(iv) Electric Power Meters

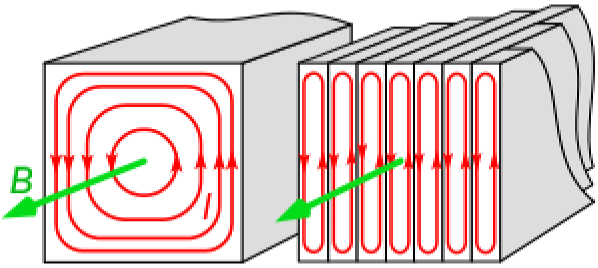
The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by the magnetic fields produced by sinusoidally varying currents in a coil. You may observe the rotating shiny disc in the power meter (analogue type) that may be linked to the 'power supply' of your house.

Disadvantages of Eddy Currents

Eddy currents generate resistive losses; they transform some forms of energy, such as kinetic energy, into heat. This Joule heating reduces efficiency of iron-core transformers and electric motors and other devices that use changing magnetic fields.

Eddy currents are minimized in these devices by selecting core materials that have low electrical conductivity (e.g., ferrites) or by using thin sheets of magnetic material, known as laminations. Electrons cannot cross the insulating gap between the laminations and so are unable to circulate on wide arcs.

The shorter the distance between adjacent laminations (i.e., the greater the number of laminations per unit area, perpendicular to the applied field), the greater the suppression of eddy currents.



source: https://i.stack.imgur.com/A1D3n.png

Eddy currents (I, red) (left one) within a solid iron transformer core (right one). Making the core out of thin laminations parallel to the field (B, green) with insulation between them reduces the eddy currents. Although the field and currents are shown in one direction, they actually reverse direction, along with the alternating current in the transformer winding.

Self Induction

An EMF or an electric current can be induced in a coil, due to the magnetic flux change produced by another coil in its vicinity or by the flux change produced by the same coil. These two situations are described separately in the next two subsections.

However, in both the cases, the flux through a coil is proportional to the current.

That is, Φ_B α I.

Further, if the geometry of the coil does not vary with time then,

$\frac{d\emptyset}{dt} \propto \frac{dI}{dt}$

For a closely wound coil of N turns, the same magnetic flux is linked with all the turns. When the flux Φ_B through the coil changes, each turn contributes to the induced emf. Therefore, a term called **flux linkage** is used which is equal to $N\Phi_B$ for a closely wound coil and in such a case:

$N\,\Phi_B \propto I$

The constant of proportionality, in this relation, is called the **self-inductance.**

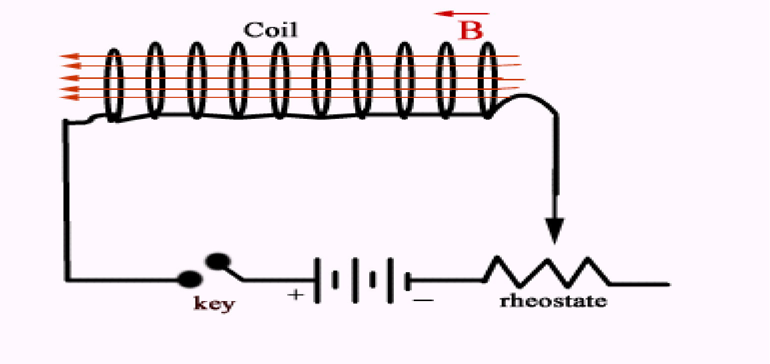
Inductance depends on:

- **(i) The geometry of the coil**
- **(ii) Intrinsic material properties of the coil and its core**

This aspect is akin to capacitance which for a parallel plate capacitor depends on the plate area and plate separation (Geometry) and the dielectric constant *K* of the intervening medium (intrinsic material property).

- **Inductance is a scalar quantity.**
- **It has the dimensions [$ML^2T^{-2}A^{-2}$] given by the dimensions of flux divided by the dimensions of current.**
- **The SI unit of inductance is henry and is denoted by H.**
- **It is named in honour of Joseph Henry who discovered electromagnetic induction in USA, independently of Faraday in England**

Self-induction is the property of a coil by virtue of which it opposes any change in the strength of current flowing through it; it does so by inducing an opposing emf upon itself.



Source: If the current flowing through the circuit is changed by adjusting the rheostat then an emf is self induced in the coil so as to resist this change. As mentioned earlier if I is the current flowing through the coil, the magnetic flux (Ø) linked with the coil is such that:

$$\varnothing \propto I$$

We can, therefore put:

$$\varnothing = LI$$

Here, L is the constant of proportionality; it is called coefficient of self induction, for the given coil.

The value of L depends on the following:

- **Number of turns**
- **Area of cross section of the coil**
- **Geometry of the coil**
- **Nature of material of the core**

If I = 1 unit, then L= ϕ

Hence,

"Coefficient of self induction for a coil, is defined as the amount of magnetic flux linked with the coil when a unit current flows through the coil itself.

Now, the emf induced in the coil is given by:

$$e = \frac{-d\phi}{dt} \frac{= -d(LI)}{dt} = -L\frac{dI}{dt}$$

$$\therefore |\varepsilon| = L\frac{dI}{dt}$$

$Hence,\ if \frac{d(I)}{dt}$ = 1unit, we have $|\varepsilon| = L$

Hence, **coefficient of self induction can also be defined as the magnitude of emf, induced in the coil, when the rate of change of current through the coil itself, is unity.**

The SI unit of self-inductance is henry (H).

$$Now,\ L = \frac{|e|}{\frac{dI}{dt}}$$

$$\therefore 1\ henry = \frac{1\ volt}{1\ ampere/s} = 1\left(\frac{volt-s}{ampere}\right) = 1(ohm)$$

Self Inductance of a Long Solenoid

A solenoid is a tightly packed helical coil; **a long solenoid is one whose length is very large compared to its radius.**

Now, we know that for a solenoid magnetic flux, linked with the coil, is given by:

ϕ = LI

⇨ LI = NBA

⇨ $LI = N\left(\frac{\mu_0 NI}{l}\right)A$

⇨ $L = \frac{\mu_0 N^2 A}{l}$

If the core of the solenoid is made of any other magnetic material then $\mu = \mu_o\mu_r$

⇨ $L = \frac{\mu N^2 A}{l}$

⇨ $L = \frac{\mu_0\mu_r N^2 A}{l}$

If n is number of turns per unit length then n = N/l

⇨ $L = \frac{\mu_0\mu_r N^2 Al}{l^2}$

⇨ $L = \mu_o\mu_r n^2 Al$

Energy Required to Build up a Current Through a Solenoid

The self-induced emf is also called the back emf as it opposes any change in the current in a circuit. Physically, self-inductance plays the role of electrical inertia.

When a current flows through a solenoid, work has to be done against the back emf. This work done is stored as magnetic potential energy. For the current, I, at any instant in a circuit, the rate of doing work is:

$P = \frac{dW}{dt} = \varepsilon I$ $\qquad [P = VI]$

$\therefore \frac{dW}{dt} = I\left(\frac{L\,dI}{dt}\right)$ $\qquad [|\varepsilon|] = L\frac{dl}{dt}$

$\therefore dW = LIdl$

The total amount of work done, in establishing a current I, is given by:

$$W = \int dW = L\int_0^I (IdI)$$

$$\therefore W = \frac{1}{2}LI^2$$

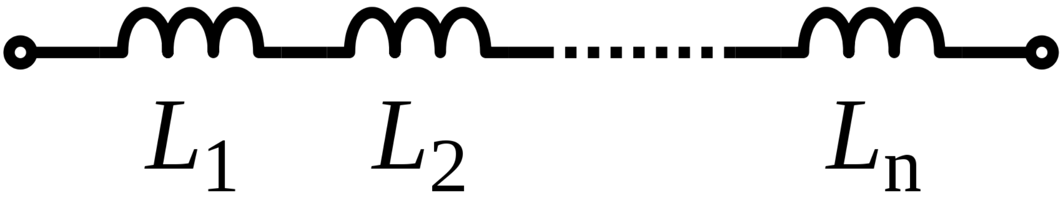
This expression is similar to the expression, **$K = \frac{1}{2}mv^2$** for the (mechanical) kinetic energy,

It suggests that L can be regarded as analogous to m.

Just as ‘mass’ (mechanical inertia) is an indicator of the ‘opposition’ to the change in state of rest or uniform motion ,velocity of an object, the self indicator (L) (electrical inertia) is an indicator of the ‘opposition’ to the change in current flowing in a circuit.

Grouping of Inductances

Coils Connected in Series



When any number of coils are connected in series, the same amount of current flows through them; the potentials across them differ and add up to the total potential applied across the combination.

Mathematically:

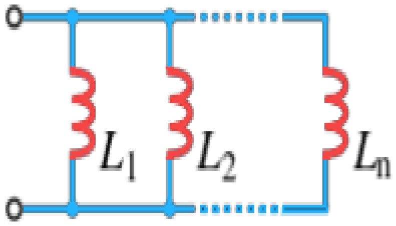
$$e = e_1 + e_2 + \ldots\ldots + e_n$$

$$\therefore L_s \frac{dI}{dt} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + L_3 \frac{dI}{dt} ------ + L_n \frac{dI}{dt}$$

$$\Rightarrow L_s = L_1 + L_2 + \ldots\ldots + L_n$$

Therefore, combined net self inductance of n coils, connected in series, is equal to the sum of the inductances of individual coils. In a series combination the total inductance increases.

Coils Connected in Parallel:



When coils are connected in parallel, the potential across all of them has the same value. The current gets divided; the total current drawn from the 'source' equals the sum of the currents through the individual coils. .

Hence, $I_p = I_1 + I_2 + \ldots\ldots\ldots + I_n$

$$\therefore \frac{dI_P}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} -----+ \frac{dI_n}{dt}$$

$$\frac{e}{L_P} = \frac{e}{L_1} + \frac{e}{L_2} + \frac{e}{L_3} + \;-----\; + \; e \qquad [as,\ potential\ is\ same\ across\ all]$$

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} ----+ \frac{1}{L_n}$$

So, in parallel connection, the reciprocal of the combined inductance of a number of coils is equal to the sum of the reciprocals of inductances of individual coils.

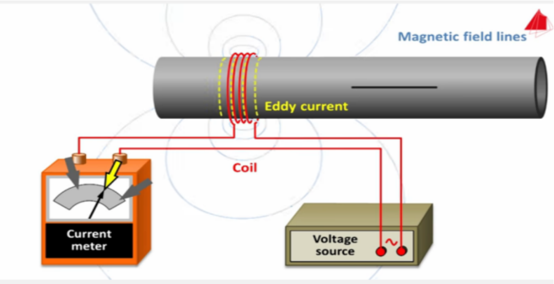
Thus , in a parallel combination the total inductance is less than the minimum value of inductance of any coil.

Check this out

One important use of self inductance is to locate material faults, cracks homogeneity etc

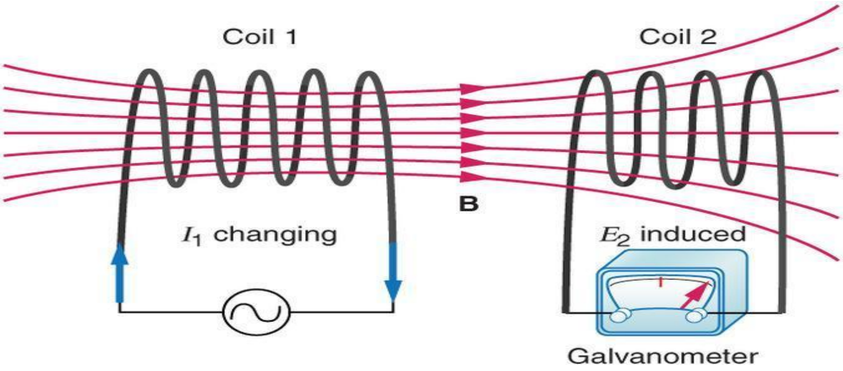
https://www.youtube.com/watch?v=oriFJByl6Hs

self induction and eddy current meters



Mutual Inductance

Let us consider the second case in which an electric current can be induced in a coil due to flux change produced by another coil in its vicinity:



Mutual Induction is a phenomenon in which a changing current in coil 1 produces a changing magnetic flux for coil 2 (placed in its field); this induces an emf in coil 2 .This emf is induced in a way so as to oppose the cause of its production.

The induced current will be induced in a direction which will oppose the increasing magnetic flux associated with the coil 2.

Coefficient of Mutual Induction

It is found that

$\phi \propto I$

Or

$\phi = MI$

M is the constant of proportionality, called **Coefficient of mutual induction**.

This is numerically equal to magnetic flux linked with one of the coil when a unit current flows through the neighbouring (second) coil.

So, $\phi = M$ when $I = 1$

The induced emf in the neighbouring coils given by:

$$\varepsilon = \frac{-d\phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$$

$$\therefore If\ \frac{dI}{dt} = 1,\ |\varepsilon| = M$$

Hence, **Coefficient of mutual induction can also be defined as the emf induced in one coil when rate of change of current through the other is unity**.

SI unit of mutual induction is henry (H).

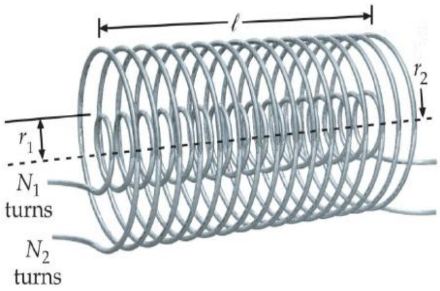
Coefficient of mutual induction is said to be one henry when a rate of change of current of one ampere/sec through one coil, induces an emf of one volt in the other coil.

Factors on which Coefficient of mutual induction depends:

- **Geometry of the two coils: size, shape, number of turns.**
- **Nature of the material of which the coils are made up of.**
- **Nature of the material on which the two coils are wound on.**
- **Distance between the coils.**
- **Relative orientation of the two coils.**

Draw a schematic diagram for each and think of an explanation why mutual induction should depend on the above factors.

Mutual Induction of two long Coaxial Solenoids



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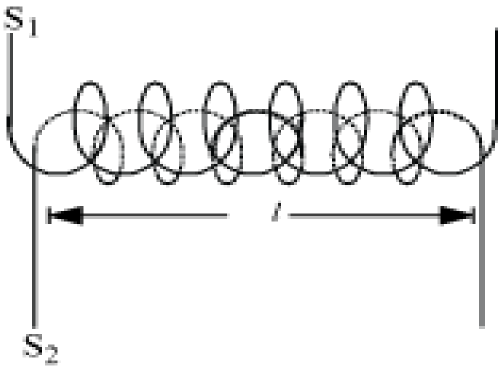


Source:
https://www.meritnation.com/ask-answer/question/deduce-the-expression-for-the-mutual-inductance-of-two-long/physics/5939145

Consider the above figure which shows two long coaxial solenoids each of length L. We denote the radius of the inner solenoid S_1 by r_1 and the number of turns per unit length by n_1. The corresponding quantities for the outer solenoid S_2 are r_2 and n_2, respectively. Let N_1 and N_2 be the total number of turns of coils S_1 and S_2, respectively.

When a current I_2 is set up through S_2, it in turn sets up a magnetic flux through S_1. Let us denote it by Φ_1. The corresponding flux linkage with solenoid S_1 is:

$N_1\phi_1 = M_{12} I_2$ ------- [1]

M_{12} is called the mutual inductance of solenoid S_1 with respect to solenoid S_2. It is also referred to as the coefficient of mutual induction.

Eq. [1] clearly indicates that a current in coil 2 induces a magnetic flux in coil 1 and M_{12} is the linkage between them.

To calculate M_{12}:

$N_1\phi_1 = N_1 B_2 A_1$

$= (n_1 L)(\mu_o n_2 I_2)(\pi r_1^2)$

$= \mu_o n_1 n_2 \pi r_1^2 L I_2$ ------------- [2]

Compare eq.[1] and eq.[2] we get

$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 L$ ---------------- [3]

We have neglected the edge effects and considered the magnetic field to be uniform throughout the length and width of the solenoid S_2. This is a good approximation when the solenoid is long, implying $L >> r_2$.

If we consider the reverse case in which current I_1 flows through S_1 and a flux of ϕ_2 gets linked with the coil 2 through mutual inductance M_{21}, the flux linkage is given by:

$N_2\phi_2 = M_{21} I_1$ ----------- [4]

M_{21} is the mutual inductance of solenoid S_2 with respect to solenoid S_1.

So, M_{21} is given by:

$N_2\phi_2 = N_2 B_1 A_1$

Here we have used A_1 instead of A_2 because the magnetic flux due to current in coil S1 is limited to S1 only as it's a long solenoid no field is there outside S1 and effective area will be A_1.

$N_2\phi_2 = N_2 B_1 A_1$

$= (n_2 L)(\mu_0 n_1 I_1)(\pi r_1^2)$

$= \mu_0 n_1 n_2 \pi r_1^2 L I_1$ ------------- [5]

Comparing eq. [4] and eq. [5]

$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 L$ -------------- [6]

From equation [3] and [6] we see that $M_{21} = M_{12} = M$

If the inner solenoid is smaller than the outer solenoid, we can find the linkage $N_1\phi_1$ easily as the inner solenoid is in the uniform magnetic field of the outer solenoid. Finding M_{12} is easy but when we try to find M_{21} then it is difficult as now the outer coil is in a non-uniform field of inner solenoid.

In such a situation $M_{21} = M_{12} = M$ is very useful.

If a medium of relative permeability μ_r had been present,

The mutual inductance would become:

$$M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 L$$

It is also important to know that the mutual inductance of a pair of coils, solenoids, etc., depends on their separation as well as their relative orientation.

Answer the following:

- **Can we have an inductance without a resistance? How about a resistance with an inductance?**

  Answer: No, as every material has some resistance. Yes, we simply have to coil a wire to have resistance with an inductor.

- **What will a change in magnetic flux induce? An emf or a current?**

  Answer: The change in magnetic flux will induce an emf always. A current will flow only when the loop or circuit is complete.

- **When is the magnetic flux linked with the coil held in a magnetic field zero?**

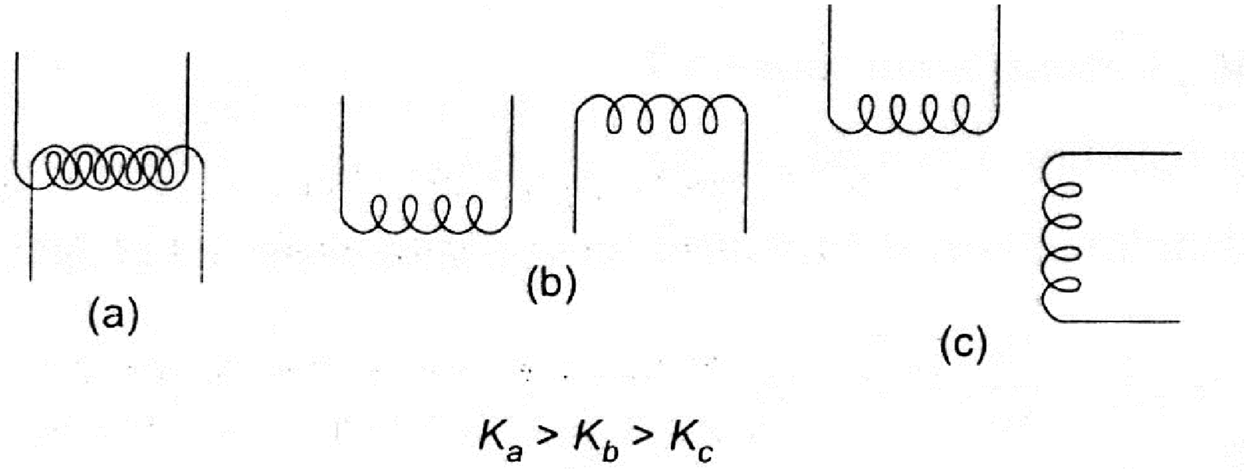
  Answer: When the plane of the coil is along the field, the magnetic flux linked with the coil held in such a magnetic field will be zero.

Coefficient of coupling *K*, of two coils, is a measure of the magnetic coupling of the two coils. It is given by

$$k = \sqrt{\frac{M}{L_1 L_2}}$$

where M is the coefficient of mutual inductance of the coils and L_1 L_2 *and L* are the coefficients of self inductance of the two coils.

K is always< 1. It can equal one only in the ideal case of perfect magnetic coupling i.e., in the case where all the field lines, of the magnetic field, produced by the current flowing in one coil, get linked with, or pass through, the second coil



Above Fig. shows a pair of coils in three different orientation, (a), (b), (c), and relative values of their coefficients of coupling.A coil, having some self inductance, is also called an inductor with inductance L.

Example

Two concentric circular coils, one of small radius r_1 and the other of large radius r_2, such that $r_1 << r_2$, are placed coaxially with centres coinciding.Obtain the mutual inductance of the arrangement.

Solution

Let a current I_2 flow through the outer circular coil. The field at the centre of the coil is

$B_2 = \mu_o I_2 / 2r_2$.

Since the other co-axially placed coil has a very small radius, B_2 may be considered constant over its cross-sectional area.

Hence,

$$\emptyset_1 = \pi r_1^2 B_2 = \frac{\mu_0 \pi r_1^2}{2r_2} I_2 = M_{12} I_2$$

Thus,

$$M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2} I_2$$

$$M_{12} = M_{21} = \frac{\mu_0 \pi r_1^2}{2r_2}$$

Note that we calculated M_{12} from an approximate value of Φ_1, assuming the magnetic field B_2 to be uniform over the area πr_1^2

However, we can accept this value because $r_1 << r_2$.

Summary

- **Eddy Currents:** When bulk pieces of conductors are subjected to a changing magnetic flux, induced currents are produced in them. The currents, which are induced in bulk pieces of conductor, when the magnetic flux linked with them changes, are called eddy currents. The flow patterns of such induced currents, (in bulk pieces of conductors), can resemble swirling eddies in water. It is for this reason that these currents are called eddy currents. This effect was discovered by physicist Foucault (1819-1868); hence these currents are also called Foucault currents.
- Magnetic flux changes whenever either the magnetic field or the area of the coil changes, or the orientation of the coil, the respect to the magnetic field changes, or a combination of the above.
- When a linear conductor moves in a magnetic field, an emf is induced across its ends.
- When a wheel rotates in a magnetic field, an emf may be induced between the center and rim of the wheel.
- Eddy currents are produced in a conductor when the magnetic flux associated with it changes with time.
- Eddy currents often cause increased power consumption, power loss and wear and tear of machine parts. They can, however, be used for several useful applications also.
- **Induction:** An electric current can be induced in a coil due to the change in magnetic flux, produced by another coil in its vicinity, or by the flux change, produced by the same coil. However, in both the cases, the flux through a coil is proportional to the current. That is, $\Phi_B \propto I$.
- **Self Induction:** Self-induction is the property of a coil by virtue of which it opposes any change in the strength of current flowing through it; it does so by inducing an (opposing) emf upon itself.
- **Coefficient of Self Inductance** of a coil, L is equal to (i) magnetic flux linked with a coil when a current of 1A flows through it (ii) *e.m.f.* induced in the coil when the current in the coil changes at the rate of 1A/s.
- The **self inductance of a long solenoid** is:

$$L = \frac{\mu_0 \mu_r n^2 A}{l}$$

where A is the area of cross section of the solenoid and l is its length. Here r is the relative permeability of the core material.

- **Mutual Induction:** Mutual Induction is a phenomenon in which a changing current in coil 1, produces a changing magnetic flux for coil 2 (placed in its field); this induces an emf in coil 2 .This emf is induced in a way so as to oppose the cause of its production. The induced current will be induced in a direction which will oppose the increasing magnetic flux associated with the coil 2.
- **Coefficient of Mutual Inductance,** between a pair of coils M is equal to
  - magnetic flux linked with one coil when current of 1 A flows through the other
  - emf induced in one coil when the rate of change of current, in the other coil is 1A/s.
- **The Mutual Inductance Between a Pair of Coils** (having a total number of turns n_1 (primary) and n_2 (secondary), is:

  $$M = \frac{\mu_0 \mu_r n_1 n_2 A}{l}$$

  A is the area of the cross section of the secondary and l is the length of the primary.
- **S.I. unit** of L or M is henry (H)